Who does the Lottery Benefit?

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Level/Course

The social justice goal of this lesson is that students consider some of the arguments against the lottery—mainly that it functions as a regressive tax and is not a profitable investment.

The mathematics of this lesson is most appropriate for a course in Discrete Math or an Algebra II class which covers combinations, permutations, and probability.

Objectives

- The learner will use data analysis to model real-world problems.
- The learner will use logic and deductive reasoning to draw conclusions and solve real-world problems.

Activities

Introduction

The lottery is a current topic of debate in North Carolina. The introduction provides that students have some understanding of the purpose of lotteries as well as one of the arguments against the lottery. The Georgia Lottery is given as an example. An extended research project in which students investigate the lottery debate in North Carolina more deeply is also recommended, though not included here.

Part I: Can playing the lottery be profitable?

Often, people think that paying one or two dollars for the chance to win hundreds of thousands or even millions of dollars is a wise investment. Students are asked to determine probabilities of winning the Fantasy 5 game in Georgia and to analyze the expected value of buying a lottery ticket.

Part II: Who is harmed the most by playing the lottery?

Students will be asked to determine the cost of playing the lottery with a "realistic" chance of winning. Then they will be asked to consider the effect that this spending would have on a families income—comparing and contrasting a family near the poverty line and a family living on Georgia's median income. Students will be asked to synthesize their findings from Parts I and II and justify an argument of whether or not the lottery functions as a regressive tax.

Assessment

For each part, students are asked to calculate or determine various probabilities and statistics. They must then use these numbers to justify claims about the lottery. Either in groups or individually, they should submit written and/or oral reports of their calculations, findings, and conclusions.

Introduction

1. What is a lottery?

Many states have adopted lotteries whose main purpose is to provide additional state funds to pay for and improve the state's education. For every lottery ticket purchased, a portion of the ticket sale goes toward the state education budget, while the remainder of the money from the ticket sale goes toward the jackpot! Usually, hundreds, thousands, even ten thousands of lottery tickets are sold everyday across any given state, creating huge jackpots. The larger the jackpot gets, the more people that are tempted to play.

2. Example: The Georgia Lottery

A. **How it works:** The state of Georgia uses the revenue generated from its lottery to provide college scholarships to its residents. The Hope Scholarship program offers tuition scholarships to any Georgia resident who has graduated from high school with at least a 3.0 GPA, with one stipulation: the recipient must attend a public institution of higher education (that is, not a private school) in the state of Georgia. The scholarship is renewable for four years provided that the student keeps a 3.0 GPA throughout college.

B. **So what's the catch?** Lotteries ideally provide extra money to improve education within a state. But many critics cite shortcomings of the lottery. Mainly, the lottery displaces wealth regressively. In other words, because poor and working class citizens are particularly attracted to the lottery due to the hope of winning huge sums of money, changing their lives and climbing the economic latter, they buy more lottery tickets than the average citizen. But the chances of winning are *extremely* small. So the money, which is put into the lottery by families who can afford it the least, gets redistributed to families who mostly do not need financial help in the form of college scholarships. In general, poor citizens pay for middle-class and even rich citizens to go to college. Of course, over time this accentuates the income gap between poor and rich, making improvement of socio-economic status from generation to generation extremely difficult. In short, the lottery destroys the American dream by "taxing the poor" and giving to the rich.

C. **Fantasy 5:** The Fantasy 5 game is one of the most popular lotto games in Georgia. You will use the details of the game to investigate this complaint—that the lottery is not a profitable investment, and it displaces wealth regressively.

Part I: Can playing the lottery be profitable?

1. Fantasy 5—How the game works

The Fantasy 5 game is one of Georgia's most popular lotto games because it allows players to choose the numbers they wish to be entered into the drawing instead of being assigned a number. This ensures that the drawing cannot be fixed! Furthermore, drawings occur everyday with lots of winners (relatively speaking).

<u>How to play</u>

On the Fantasy 5 card, the player selects 5 numbers, all between 0 and 39. Numbers cannot be repeated. Then the player pays $1 for every time he/she wishes the chosen numbers to be entered in a drawing.

For example, a player selects the numbers 8, 14, 17, 22, and 31. He wants these numbers to be entered in 30 drawings. He must fill out the lotto card with his chosen numbers and then pay $30—one dollar for each drawing entered.

<u>How to win</u>

Five numbers are drawn at random. Prizes are awarded in the following manner:

Match	Prize Awarded
0 or 1	No prize
2	1 free play
3	3rd Prize
4	2nd Prize
5	1st Prize

The prize amounts are determined by how many players buy tickets each day. If no player wins the first prize, then the money which would have been awarded rolls over to the next day's 1st prize pool. This way, if there is no winner, the jackpot continues to increase.

2. Exercises

a. For the Fantasy 5 lotto game, find:

i. The number of ways to draw the 5 random winning numbers

ii. The number of ways to match all 5

iii. The number of ways to match 4

iv. The number of ways to match 3

v. The number of ways to match 2

vi. The number of ways to match 1 or 0.

b. Calculate the probabilities to match 0 or 1 (one event because both mean you lose!), 2, 3, 4, and 5. Fill in the table.

Matched	Probability
0 or 1	
2	
3	
4	
5	

c. Use Table I to find the average payout for each of the winning events (matching 3, 4, and 5 of the numbers). Then, determine how much you win or lose on average in each of the five possibilities (matching 0 or 1, 2, 3, 4, or 5) by multiplying the amount you would win in a given event by the probability that the event occurs (Hint: Remember that you paid $1, regardless of whether or not you win. Also, because matching 2 wins you a free play, consider this event the same as having a payout of $1).

d. Now using your results, determine the expected value for one play (entering one drawing for $1) of the Fantasy 5 lottery. How much on average will you win (or lose) each time you play Fantasy 5? Using your calculations to justify your answer, explain whether or not the lottery is a profitable investment.

Table I

Average Payouts for Fantasy 5 Lotto Game			
	3 of 5	4 of 5	5 of 5
Payout Amounts	$ 19.00	$ 400.00	$ 33,930.00
	$ 20.00	$ 515.00	$ 59,278.00
	$ 21.00	$ 514.00	$ 101,668.00
	$ 20.00	$ 399.00	$ 217,866.00
	$ 20.00	$ 377.00	$ 330,980.00
	$ 16.00	$ 314.00	$ 269,088.00
	$ 23.00	$ 571.00	$ 25,718.00
	$ 14.00	$ 249.00	$ 91,117.00
	$ 19.00	$ 399.00	$ 197,930.00
	$ 19.00	$ 335.00	$ 53,626.00
	$ 20.00	$ 467.00	$ 198,598.00
	$ 14.00	$ 251.00	$ 118,056.00
	$ 15.00	$ 328.00	$ 269,565.00
	$ 19.00	$ 457.00	$ 260,211.00
	$ 20.00	$ 502.00	$ 314,855.00
	$ 17.00	$ 324.00	$ 206,544.00
	$ 15.00	$ 368.00	$ 173,966.00
	$ 17.00	$ 297.00	$ 564,898.00
	$ 21.00	$ 508.00	$ 261,819.00
	$ 20.00	$ 427.00	$ 311,253.00
	$ 16.00	$ 291.00	$ 670,538.00
	$ 16.00	$ 292.00	$ 56,257.00
	$ 22.00	$ 438.00	$ 88,735.00
	$ 14.00	$ 243.00	$ 105,256.00
	$ 17.00	$ 452.00	$ 696,461.00
	$ 17.00	$ 314.00	$ 57,896.00
	$ 15.00	$ 318.00	$ 316,216.00
	$ 15.00	$ 276.00	$ 218,869.00
	$ 19.00	$ 405.00	$ 33,930.00
	$ 20.00	$ 482.00	$ 59,278.00
	$ 19.00	$ 417.00	$ 118,056.00
	$ 13.00	$ 303.00	$ 260,211.00
	$ 13.00	$ 282.00	$ 314,855.00
	$ 16.00	$ 326.00	$ 173,966.00
	$ 16.00	$ 395.00	$ 173,966.00
	$ 17.00	$ 476.00	$ 173,966.00
	$ 16.00	$ 343.00	$ 88,735.00
	$ 17.00	$ 376.00	$ 105,256.00
Averages	***$ 17.55***	***$ 379.76***	***$204,563.50***

Source: The Georgia Lottery Corporation

Part II: Who is harmed the most by playing the lottery?

1. How much does it cost to play the lottery well?

a. Suppose you want to give yourself a realistic shot at winning a decent prize from the lottery. Because the average payout for matching 3 numbers is only $17.55, hardly life-changing, you decide this prize is not even worth shooting for. Therefore, you decide that you want to give yourself a 30% shot at winning the first or second prize. In other words, you want to ensure you have a 30% chance to match at least 4 of the numbers. How many times do you need to play Fantasy 5 to ensure a 30% chance of winning the first or second prize? How much would this cost?

b. Now suppose that 2 families of 4, Family A and Family B, are playing Fantasy 5 as outlined above. That is, they want to ensure a 30% chance of winning first or second prize. Family A has a median income at the poverty line, and Family B has an income equal to the median income for families of four in Georgia. Use Tables II-A and II-B to determine the income for both families. In terms of percentage, how much more must Family A spend to play Fantasy 5 than Family B?

c. How many times must you play Fantasy 5 to ensure a 30% chance of winning first prize? What about just 5%?

2. Where do the winners live?

a. Table II-C lists the cities and towns in Georgia where the most recent first prize tickets were sold. Determine the average median income for a city where winning tickets were sold. (Hint: don't forget to include cities/towns that sold more than one ticket the appropriate number of times).

b. Next, compare the average median income for towns with winning tickets to the median income for families in Georgia. You can find data on Georgia family incomes in Table II-A. Is the average median income of towns with lottery winners more or less than the median income of Georgia families?

c. Using your calculations from Part I: Can the lottery be profitable?, from the exercises above about ensuring a 30% success rate, and your observations about median incomes in Georgia, comment on whether or not the lottery is a "tax on the poor." Argue whether or not the lottery benefits or harms citizens. Use math to justify your claims!

Table II-A

Georgia Median Family Income			
Georgia	Estimate	Lower Bound	Upper Bound
Total:	49745	48971	50519
2-person families	45775	44133	47417
3-person families	49855	47768	51942
4-person families	58060	54857	61263
5-person families	53908	51637	56179
6-person families	52947	44477	61417
7-or-more-person families	51928	46674	57182

Given in 2004 inflation adjusted dollars
Source: US census bureau

Table II-B

2004 Poverty Line	
Size of Family Unit	**Weighted Average Thresholds**
1-person families	9645
2-person families	12334
3-person families	15067
4-person families	19307
5-person families	22831
6-person families	25788
7-person families	29236
8-person families	32641
9-or-more-person families	39048

Source: US census bureau

Table II-C

Median Incomes of Cities/Towns where Winning Tickets were Sold			
City	**# First Prizes**	**Median Income**	**(Median Income) x (# First Prizes)**
Alpharetta	1	71207	71207
Ashburn	1	18702	18702
Athens	1	28403	28403
Atlanta	2	32635	65270
Augusta	1	37194	37194
Austell	1	38933	38933
Cadwell	1	32727	32727
Cohutta	1	41563	41563
Columbus	1	34798	34798
Cumming	1	38237	38237
Dalton	1	34312	34312
Doraville	1	40641	40641
Douglasville	1	45289	45289
Fayetteville	1	55208	55208
Flowery Branch	2	35478	70956
Forest Park	1	33556	33556
Griffin	1	30088	30088
Helen	1	32917	32917
Jackson	1	28472	28472
Jenkinsburg	1	40417	40417
Kingsland	1	41303	41303
Lyons	1	21202	21202
McDonough	1	41482	41482
Morrow	1	46569	46569
Moultrie	1	22193	22193
Peachtree City	1	76458	76458
Rentz	1	25000	25000
Rex	2	52714	105428
Stone Mountain	1	38603	38603
Suwanee	1	84038	84038
Sylvester	1	24114	24114
Trenton	1	34612	34612
Tucker	1	59953	59953
West Point	1	31886	31886
Totals:	***37***	***1350904***	***1471731***

Source: US Census Bureau & The Georgia Lottery Corporation

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Solutions

Part I
2. a.

i. # of ways to draw 5 numbers from 39, without repeating:

$${}_{39}C_5 = \frac{39!}{5!(39-5)!} = \frac{39!}{5!(34!)} = \frac{39 \cdot 38 \cdot 37 \cdot 36 \cdot 35}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 575{,}757$$

ii. There are 5 “good” numbers and 34 “bad” numbers. # of ways to match all five—must choose all 5 “good” numbers and 0 “bad” numbers:

$({}_5C_5)\ ({}_{34}C_0) = (1)(1) = 1$

iii. # ways to match 4:

$({}_5C_4)\ ({}_{34}C_1) = 170$

iv. # ways to match 3:

$({}_5C_3)\ ({}_{34}C_2) = 5610$

v. # ways to match 2:

$({}_5C_2)\ ({}_{34}C_3) = 59840$

vi. Matching 1 and matching 0 are mutually exclusive events, So the # of ways to match 0 *or* 1 will be the # ways to match zero *added* to the # of ways to match 1:

$({}_5C_0)\ ({}_{34}C_5) + ({}_5C_1)\ ({}_{34}C_4) = 287{,}256 + 231{,}880 = 510{,}136$

2.b. The probability to match 0 or 1 is given by:

$$P(0 or 1) = \frac{(\#of\ ways\ to\ match\ 0) + (\#of\ ways\ to\ match\ 1)}{\#\ ways\ to\ choose\ 5\ from\ 39\ w/out\ repeating} = \frac{510{,}136}{575{,}757} \approx 0.886$$

To match 2:

$$P(2) = \frac{59840}{575757} \approx 0.104\,;$$

and so on for matching 3, 4, and 5.

The completed table should be:

Matched	Probability
0 or 1	0.886
2	0.104
3	0.00974
4	0.000295
5	1.73×10^{-6}

2.c.

-When you match 0 or 1, you lose $1. You spent $1 and won nothing. Then, in this event, you win:

(-$1)(0.886) = - $ 0.886

-When you match 2, you break even. You spent $1, and you won $1. Then, in this event, you win:

($0)(0.104) = $0

-When you match 3, you win (on average) $16.55. You spent $1, and you won $17.55. Then, in this event, you win:

($16.55)(0.00974) = $ 0.161

-When you match 4, you win (on average) $378.76. You spent $1, and you won $379.76. Then, in this event, you win:

($378.76)(0.000295) = $ 0.112

-When you match 5, you win (on average) $204,562.50. You spent $1, and you won $204,563.50. Then in this event, you win:

($204,563.50)($1.73 \times 10^{-6}$) = $ 0.354

2.d. The expected value is defined as: $\sum_x xP(x)$.

We've already calculated all the terms of the sum. So the expected value is:

EV = $0.354 + $0.112 + $0.161 + $0 + (-$0.886) = -$0.259

Because of the negative expected value for a given play, on average, you can expect a return of -$0.26 every time you spend $1 on the lottery. Therefore, the lottery is *not* a good investment because you should never expect to win money in the long run. In fact, if you played 100 times, you should expect to lose 100 x $0.26 = $26 in terms of equity, not to mention the $100 you lost in buying tickets.

Part II

1.a. The probability to match 4 or 5 is given by:

$P(4)+P(5)=2.95\times10^{-4}+1.73\times10^{-6}=2.97\times10^{-4}$.

Then, the number of times you need to play the lottery to give yourself a 30% chance is given by

$n=\dfrac{0.30}{P(4)+P(5)}=\dfrac{0.30}{2.97\times10^{-4}}\approx1011$, because each play is independent.

1.b. Each play costs $1, so it would cost $1011 to get a 30% chance. For Family A, this represents

$\dfrac{1011}{19307}\approx0.052=5.2\%$ of their yearly income.

For Family B, this represents

$\dfrac{1011}{58060}\approx0.017=1.7\%$ of their yearly income.

Then Family A spends

$5.2-1.7=3.5\%$

more of their income to have a realistic shot at winning. Or, Family A spends

$5.2/1.7\approx3.06$ times more of their income, in terms of percentage, than Family B.

1.c. $n=\dfrac{0.30}{1.73\times10^{-6}}=173{,}410$ to give a 30% chance.

To give a 5% chance, you must still play

$n=\dfrac{0.05}{1.73\times10^{-6}}\approx28902$ times.

2.a. To get the average median income we must divide the total in column 3 of Table II-C by 37.

$\dfrac{\$1{,}471{,}731}{37}\approx\$39{,}776$

2.b. The median income for families of 4 in Georgia is $58,060 and the average median income for towns where lottery winners were located is $39,776—a difference of $18284. We could assert that there is something systematically different in terms of income between the population of lottery players and the total population of Georgia families.

2.c. Students should touch on the negative expected value, the higher percentage of income that poor families must contribute to have a decent probability of winning (i.e. it acts as a regressive tax), and the fact that the population of lottery players appears to have a lower income than the whole population of Georgia families.